Chapter 2: Kinematics, the math of motion

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Chapter 2: Kinematics, the math of motion

- 2.1 Uniform motion p. 34-38
Chapter 2: Kinematics, the math of motion

The simplest form of motion is **uniform motion**

A motion is uniform if its position-versus-time graph is a straight line.

---

**Uniform motion**

- The displacements between successive frames are the same. Dots are equally spaced. \( v_x \) is constant.

**Nonuniform motion**

- The displacements between successive frames are not the same. Dots are not equally spaced. \( v_x \) is not constant.

---

**Position graph**

- Uniform motion: Position graph is a straight line. The slope of the line is \( v_{\text{avg}} \).
- Nonuniform motion: Position graph is curved.
Chapter 2: Kinematics, the math of motion

For uniform motion the (average) velocity remains constant:

\[ v_x = \frac{\Delta x}{\Delta t} = \text{const.} \]

An alternative way to say “uniform motion is described by a straight line in the position-versus-time graph”, is to say that for uniform motion \( x(t) \) is a linear function of \( t \):

\[ x(t) = x(0) + v_x t \]

\( x(0) \) is \( x(t=0) \)

Note the units:

Meters per Second = m/s

This means “\( x \)” as a function of time “\( t \)” NOT “\( x \)” times “\( t \)” as in \( a(b+c)=ab+ac! \)

Recall: the equation of a straight line is \( y(x) = ax + b \)

\( a \) = slope \( \quad b \) = y-intercept

\( x(0) \) is \( x(t=0) \)
This is even true for uniform motion in a general direction (say $s$; not necessarily along the x axis)

**Example; along an inclined plane!**

I can define a new variable

$s = (X, Y)$ such that $s_1 = (X_1, Y_1); s_2 = (X_2, Y_2); s_3 = (X_3, Y_3) \ldots$ etc ...

so that the distance of the object from the origin of my co-ordinate system is

$$s = \sqrt{x^2 + y^2}$$
Chapter 2: Kinematics, the math of motion

If $s_i \ [\text{meaning } (x_i, y_i)]$ is the initial component of the position vector in the s-direction, and $s_f \ [\text{meaning } (x_f, y_f)]$ its final position after uniform motion for a time $\Delta t$, then

$$s_f = s_i + v_s \Delta t \quad , \quad v_s = \frac{\Delta s}{\Delta t} = \frac{s_f - s_i}{t_f - t_i}$$

The slope of the line is $v_s = \frac{\Delta s}{\Delta t}$.

with $\Delta t = t_f - t_i$ and $v_s$ the velocity component along the s-axis
Cliction 2.1

Which position-versus-time graph represents the motion shown in the motion diagram below?
Chapter 2: Kinematics, the math of motion

Tactic box 2.1 (p.36): Interpreting position-versus-time graphs

- Steeper slopes correspond to faster speeds

- Negative slopes correspond to negative velocities, i.e., to motion to the left (or down)

- The slope is a ratio of intervals, not a ratio of coordinates. That is, the slope is NOT simply $x/t$ (x divided by t)

- Note that position-versus-time graphs have units on their axes.

- The “rise” $\Delta x$ is some number of meters; the “run” $\Delta t$ is some number of seconds. The physically meaningful rise and run include units, and the ratio of these units gives the units of the slope, i.e., the velocity.
Chapter 2: Kinematics, the math of motion

Practice Problem
(use units of miles and hours)

- Susan’s speed and velocity
- Bob’s speed and velocity
- The equation that describes Susan’s position-versus-time (in this case, x-vs-t) graph of motion.
- The equation that describes Bob’s position-versus-time (in this case, x-vs-t) graph of motion
- Finally, use (it means equate) the two equations to find the exact co-ordinates of the point of meeting.

Recall: the equation of a straight line is \( y(x) = ax + b \)

slope The y-intercept when \( x=0 \)
Chapter 2: Kinematics, the math of motion

- 2.2 Instantaneous velocity  p. 38-44
Suppose a physicist (disguised as a police officer) pulls you over and says,

“I just clocked you going 80 miles per hour”.

You might respond,

“But that’s impossible. I have only been driving for 20 minutes, so I can’t possibly have gone 80 Miles.”

He replies (and that’s when you suspect that you are dealing with a physicist)

“I mean at the instant I measured your velocity, you were moving at rate such that you WOULD cover a distance of 80 miles IF you were to continue at that velocity without change for an hour.”

You try to be smart:

“The law does not specify if the limit applies for AVERAGE or INSTANTANEOUS velocity”

The police who has obviously taken PHYS221 adds

“That will be an INSTANT $200 fine!”
A very important notion is that of instantaneous quantities like the instantaneous velocity.

We defined the velocity as \( v = \frac{\Delta s}{\Delta t} \), but this is only the average velocity during \( \Delta t \).

To get the instantaneous velocity let the time interval between two positions decrease, or \( \Delta t \to 0 \).
As $\Delta t \to 0$ we also have $\Delta s \to 0$

$s_f = s_i + v_s \Delta t$ , $v_s = \frac{\Delta s}{\Delta t} = \frac{s_f - s_i}{t_f - t_i}$

We have a dilemma!
The way out of this dilemma is to use limits:

\[ \frac{ds}{dt}(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \left( s(t + \Delta t) - s(t) \right) \]

This means as time interval approaches zero!

This means the instantaneous velocity at time \( t \) is the derivative of the position \( s(t) \) with respect to time \( t \).
Cliction 2.2

Which velocity-versus-time graph goes with this position-versus-time graph on the left?

(a)  
(b)  
(c)  
(d)
Before applying what we have learned about the instantaneous velocity, we should first do some **CALCULUS** involving **DERIVATIVES**: 

This is a “dummy name”. In other words we could have used $f(t)$ or $x(t)$!

“$c$” and “$n$” are constants; meaning fixed numbers

$$u(t) = c t^n \Rightarrow \frac{du}{dt} = n c t^{n-1}$$

$$\frac{d}{dt} \left( u(t) + w(t) \right) = \frac{du}{dt} + \frac{dw}{dt}$$

PRACTICE using Textbook examples 2.5 and 2.6!
Practice Problem
(use SI units)

\[ u(t) = ct^n \Rightarrow \frac{du}{dt} = n \cdot ct^{n-1} \]

\[
\frac{d}{dt} \left( u(t) + w(t) \right) = \frac{du}{dt} + \frac{dw}{dt}
\]

You are given the position-versus-time graph using:

\[ s(t) = 2t^2 \text{ m} \]

\[ u = s \]

\[ c = 2 \]

Derive the corresponding velocity-versus-time graph.
Example:

\[ x = 2t^2 \]

\[ v_x = \frac{dx}{dt} = 2 \cdot 2t^2 - 1 = 4t. \]
Chapter 2: Kinematics, the math of motion

Example:

\[ x = (-t^3 + 3t) \text{ m} \]

\[ v = (-3t^2 + 3) \text{ m/s} \]

Motion diagram:

Position at \( t = 3 \text{ s} \).
The particle is continuing to speed up to the left.

Value = 0 m/s

Turn at \( t = -1 \text{ s} \)
Moving to the left, because \( v_x < 0 \), and slowing.

The velocity is positive between \( t = -1 \text{ s} \) and \( t = 1 \text{ s} \).

Slope = -9 m/s

Slope = 0 m/s

Value = -9 m/s
Chapter 2: Kinematics, the math of motion

Example:

\[ x = \left(-t^3 + 3t\right) \text{ m} \]

\[ v = \left(-3t^2 + 3\right) \frac{\text{m}}{\text{s}} \]

Motion diagram:

- Position at \( t = 3 \text{ s} \).
  The particle is continuing to speed up to the left.

- Turn at \( t = -1 \text{ s} \).
  Moving to the left, because \( v_x < 0 \), and slowing.

- Turn at \( t = 1 \text{ s} \).
  The velocity is positive between \( t = -1 \text{ s} \) and \( t = 1 \text{ s} \).
The slope at a point on a position-versus-time graph of an object is
A. the object’s speed at that point.
B. the object’s average velocity at that point.
C. the object’s instantaneous velocity at that point.
D. the object’s acceleration at that point.
E. the distance traveled by the object to that point.
Chapter 2: Kinematics, the math of motion

Cliction

2. The area under a velocity-versus-time graph of an object is
   A. the object’s speed at that point.
   B. the object’s acceleration at that point.
   C. the distance traveled by the object.
   D. the displacement of the object.
   E. This topic was not covered in this chapter.
Chapter 2: Kinematics, the math of motion

End of Week 2
Chapter 2: Kinematics, the math of motion

- 2.3 Finding position from velocity  p. 44-48
We are now dealing with the reverse problem: finding the position $s(t)$ of a particle when we know its velocity $v_s(t)$. In other words,

$$v_s(t) = \frac{ds}{dt} \quad \Rightarrow \quad s(t_f) = s(t_i) + \int_{t_i}^{t_f} dt \; v_s(t)$$

This integral is the area between the function $v_s(t)$ and the $t$-axis.
Chapter 2: Kinematics, the math of motion

\[ s_f = s_i + \text{area under the velocity curve } v_s \text{ between } t_i \text{ and } t_f \]

\[ \Rightarrow s(t_f) = s(t_i) + \int_{t_i}^{t_f} \ dt \ v_s(t) \]

During step \( k \), the product \( \Delta s_k = (v_s)_k \Delta t \) is the area of the shaded rectangle.

This symbol means ADD all of the area of RECTANGLES as you go from \( t_i \) to \( t_f \).

Integral = Sum(Rectangles) = Sum(\( \Delta t \cdot v_i \))

NOTE:
The units of \( \Delta t \cdot v_i \) are [seconds].[meters/seconds] which yields [meters]
Chapter 2: Kinematics, the math of motion

We should now recall some **CALCULUS** involving **INTEGRALS** (inverse operation of differentiation):

This is a definite integral because there are two definite boundaries for the area we want to find.

\[
\int_{t_i}^{t_f} u \, dt = \int_{t_i}^{t_f} ct^n \, dt = \frac{ct_i^{n+1}}{n+1} - \frac{ct_f^{n+1}}{n+1} \quad (n \neq -1) \quad (2.12)
\]

\[
\int_{t_i}^{t_f} (u + w) \, dt = \int_{t_i}^{t_f} u \, dt + \int_{t_i}^{t_f} w \, dt \quad (2.13)
\]

The Integral of a sum is the sum of the Integrals!
Chapter 2: Kinematics, the math of motion

Practice Problem
(use SI units)

This is the velocity-versus-time diagram of a drag racer.

Given the velocity-versus-time graph above, what is the position-versus-time graph?
Solution: 

\[ x(t) = x(0) + \int dt \, 4t \]

\[ = x(0) + 4 \frac{t^2}{2} = 2t^2. \]

QUESTION

How far does the racer move during the first 3 seconds?

\[ X(3) = 2.3^2 = 2.9 = 18 \text{ meters}. \]
Which position-versus-time graph goes with this velocity-versus-time graph on the left? The particle’s position at \( t_i = 0 \) s is \( x_i = -10 \) m.
2.4 Motion with constant acceleration

p. 48-54

To say that “an object has a uniform acceleration of 3 m/s” is equivalent to saying that “its velocity increases by 3 m/s every second”.
Chapter 2: Kinematics, the math of motion

We now consider the second (of two) important types of motion: **motion with constant acceleration**. Remember that acceleration is the change of the velocity during a time interval $\Delta t$,

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t} \quad \text{or} \quad \Delta \vec{v} = \vec{a} \Delta t$$

An object whose velocity-versus-time graph is a **STRAIGHT LINE** has a steady and unchanging acceleration.

For a uniformly accelerated motion, the **SLOPE** of the velocity-versus-time graph (that is the acceleration) is **CONSTANT**!

The Porsche reaches 30 m/s in 6 s. The VW takes 15 s.

**Porsche**

- Slope = $a_{\text{Porsche \ avg}} = 5.0 \text{ (m/s)/s}$
- $\Delta t = 5 \text{ s}$
- $\Delta v_s = 10 \text{ m/s}$

**VW**

- Slope = $a_{\text{VW \ avg}} = 2.0 \text{ (m/s)/s}$
- $\Delta t = 5 \text{ s}$
- $\Delta v_s = 10 \text{ m/s}$
Chapter 2: Kinematics, the math of motion

If \( \vec{a} \) is constant then the velocity is a linear function of time:

The area under the straight line can be divided into rectangles. Adding the areas of these gives you the object’s velocity in time!

Final velocity

Initial velocity

\[
v_{f_x} = v_{i_x} + \text{area under the acceleration curve } a_x \text{ between } t_i \text{ and } t_f
\]

\[
v_x(t_f) = v_x(t_i) + a_x \cdot (t_f - t_i)
\]
Chapter 2: Kinematics, the math of motion

This is probably the most important equation in Phys 221. Hints to remember it:

- $v_x$ varies linearly $\rightarrow$ its integral $x$ varies quadrat.
- Check the units: to get a length, $v_x$ needs to be multiplied with $\Delta t$, $a_x$ with $\Delta t^2$
- $a_x/2$ so that it becomes $a_x$ after differentiation.
Chapter 2: Kinematics, the math of motion

TABLE 2.3 The kinematic equations for motion with constant acceleration

\[
\begin{align*}
v_{fs} &= v_{is} + a_s \Delta t \\
s_f &= s_i + v_{is} \Delta t + \frac{1}{2}a_s (\Delta t)^2 \\
v_{fs}^2 &= v_{is}^2 + 2a_s \Delta s
\end{align*}
\]

(a) Motion at constant velocity

(b) Motion at constant acceleration

The velocity is constant.  
The slope is \( v_s \).

The acceleration is constant.  
The slope is \( a_s \).

Put \( a = 0 \) in these equations

Keep \( a = \text{constant} \) in these equations
Cliction 2.4

Which velocity-versus-time graph or graphs goes with this acceleration-versus-time graph? The particle is initially moving to the right and eventually to the left.
Chapter 2: Kinematics, the math of motion

2.5 Free Fall  p. 54-57

EXPERIMENT

Hold your phys221 text book (or any book) in one hand and a sheet of paper of similar size (or smaller) in the other hand.
Drop them from the same height and at the same time.
Which one reaches the floor first?

Then, hold the same book but now put the sheet of paper on top of it, and drop them from the same height as before.
Which one reaches the floor first?

WHAT DO YOU CONCLUDE?
Free fall is the first natural constant acceleration case to study:

If air resistance can be neglected, and if we stay relatively close (50km) to the ground, all objects experience the same constant acceleration and points downward:

\[ \vec{a}_{\text{free fall}} = (9.80 \text{ m/s}^2, \text{vertically downward}) \]
\[ = (0, -9.80 \text{ m/s}^2) \]

or

\[ a_y = -9.80 \text{ m/s}^2 = -g \]

**IMPORTANT!!**

- \( g \) is NOT “gravity”!
- \( g \) is “the acceleration due to gravity”

**IMPORTANT!!**
Chapter 2: Kinematics, the math of motion

(a) The free-fall acceleration is constant. The graph has a constant slope.

(b) $a_y = -9.8 \text{ m/s}^2$
Example: How long does a falling rock takes to reach the ground when starting to fall at $y_0 = 100$ m

Physical representation

Pictorial representation

Known

- $y_0 = 100$ m
- $v_{0y} = 0$ m/s
- $t_0 = 0$ s
- $y_1 = 0$ m
- $a_y = -g = -9.8$ m/s$^2$

Find

- $t_1$ and $v_{1y}$

\[
y_1 = y_0 + v_y(0) \Delta t + \frac{a_y}{2} \Delta t^2 = y_0 - \frac{g}{2} \Delta t^2
\]

\[
\Rightarrow \Delta t = \sqrt{\frac{2(y_1 - y_0)}{g}} = \sqrt{\frac{2 \cdot 100 \text{ m}}{9.80 \text{ m/s}^2}} = (\pm) 4.52 \text{ s}
\]

NOTE:

$V_y(0) = V_{0y} = 0$ m/s
Chapter 2: Kinematics, the math of motion

2.6 Motion on an inclined plane   p. 57-61
Chapter 2: Kinematics, the math of motion

Motion on an inclined plane is also an example of uniformly accelerated motion.

The tricky part is to identify the relevant component of the Earth's acceleration.

This piece of $\vec{a}_{\text{free fall}}$ accelerates the object down the incline.
Chapter 2: Kinematics, the math of motion

Trick to simplify description: choose suitable coordinate system

We then only need to consider $x(t)$.

Components of acceleration in the tilted coordinate system:

- $a_x = a_\parallel = g \sin(\theta)$
- $a_y = a_\perp = -g \cos(\theta)$
Chapter 2: Kinematics, the math of motion

The inclined plane prevents the object to move in y-direction, we therefore have \( y(t) = 0 \) for all times.

The motion along the x-axis is now subject to the constant acceleration \( a_x = g \sin(\theta) \)

\[
x(t) = x(t_0) + v_x(t_0) \cdot (t - t_0) + \frac{g}{2} \sin(\theta) \cdot (t - t_0)^2
\]
The ball rolls up the ramp, then back down. Which is the correct acceleration graph?
Chapter 2: Kinematics, the math of motion

Practice Problem
(use SI units)

An amusement park shoots a car up a frictionless track inclined at 30°. The car rolls up, then rolls back down. If the height of the track is 20 m, what is the maximum allowable speed with which the car can start?

First figure out the sign of the acceleration \(a_x\).
Then, figure out which one of the 3 equations listed in table above is applicable here.
(see detailed solution in page 63/64 in Textbook)

<table>
<thead>
<tr>
<th>TABLE 2.3 The kinematic equations for motion with constant acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v_{fs} = v_{is} + a_s \Delta t)</td>
</tr>
<tr>
<td>(s_f = s_i + v_{is} \Delta t + \frac{1}{2}a_s (\Delta t)^2)</td>
</tr>
<tr>
<td>(v_{fs}^2 = v_{is}^2 + 2a_s \Delta s)</td>
</tr>
</tbody>
</table>

This is the table listing the known quantities.
Chapter 2: Kinematics, the math of motion

The one-dimensional acceleration along an incline is

$$a_x = \pm g \sin(\theta)$$

The CORRECT SIGN depends on the direction in which the ramp is tilted!

$$a_x = -g \sin(\theta)$$

$$a_x = +g \sin(\theta)$$

**IMPORTANT!**

**NOTE**

$g$ is the length, or magnitude, of the acceleration due to gravity

$g = 9.80\ \text{m/s}^2$

This piece of $\vec{a}_{\text{free fall}}$ accelerates the object down the incline.
Chapter 2: Kinematics, the math of motion

IMPORTANT!

\[ V_f = V_i + 2a \Delta x \]

\[ a = -g \sin(\theta) < 0 \]

\[ \Delta x = x_{\text{final}} - x_{\text{initial}} > 0 \]

\[ V_i^2 = -2a \Delta x > 0 \]

\[ V_i^2 = -2a \Delta x > 0 \]

\[ V_f = V_i + 2a \Delta x \]

\[ a = +g \sin(\theta) > 0 \]

\[ \Delta x = x_{\text{final}} - x_{\text{initial}} < 0 \]

\[ V_i^2 = -2a \Delta x > 0 \]
Chapter 2: Kinematics, the math of motion

2.7 Instantaneous acceleration  p. 61-62
Chapter 2: Kinematics, the math of motion

Also useful to understand the concepts of motion is: **instantaneous acceleration**
This can be introduced in exactly the same way as instantaneous velocity: $a_x$ is the change of $v_x$ during a time interval $\Delta t$.

Let $\Delta t$ go to zero

Thus, acceleration is the time derivative of velocity
Chapter 2: Kinematics, the math of motion

IMPORTANT
Comparing *instantaneous velocity* with *instantaneous acceleration*

---

**Diagram 1**

**Step (a)**
- Slope is maximum at B. This is the point of maximum speed.
- Slope is zero at A and C, so the speed is zero. A particle can't be any slower than \( v = 0 \).
- The slope is negative before A, so \( v_y < 0 \).

**Diagram 2**

**Step (a)**
- Turning points are where \( v_y = 0 \).
- The velocity is maximum at the instant \( a_y = 0 \).
Rank in order, from largest to smallest (amplitude), the accelerations $a_A - a_C$ at points A – C.

1) $a_A > a_B > a_C$
2) $a_C > a_A > a_B$
3) $a_C > a_B > a_A$
4) $a_B > a_A > a_C$
Example: encounter of two objects

A BMW starts at $x_{BMW}(0) = 0m$ (we set the origin at $x=0m, t=0s$) and accelerates with $a_x = g$. When does it overtake a light beam that started at the same position and has constant velocity $c = 2.99 \times 10^8$ m/s? Is this reasonable?

**TABLE 2.3** The kinematic equations for motion with constant acceleration

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{f} = v_{i} + a_{s} \Delta t$</td>
<td>Final velocity</td>
</tr>
<tr>
<td>$s_{f} = s_{i} + v_{i} \Delta t + \frac{1}{2} a_{s} (\Delta t)^2$</td>
<td>Final position</td>
</tr>
<tr>
<td>$v_{f}^2 = v_{i}^2 + 2a_{s} \Delta s$</td>
<td>Final velocity squared</td>
</tr>
</tbody>
</table>

**Constant velocity** (or $a_s = 0$)

\[
s = s(t_0) + V \cdot (t - t_0)
\]

\[
s = 0 + V \cdot (t - 0)
\]

\[
s = V \cdot t
\]
Chapter 2: Kinematics, the math of motion

Solution:

\[ X(t) = x(t_0) + v(t_0)(t - t_0) + \frac{1}{2} g (t - t_0)^2 \]

BMW: \[ x = \frac{1}{2} g t^2 \]

Light: \[ x = c t \]

\[ x_{BMW}(t_{enc}) = x_{Light}(t_{enc}) \]

\[ \frac{1}{2} g t_{enc}^2 = c t_{enc} \]

\[ t_{enc} = 0 \text{ s (start)} \quad \text{or} \quad t_{enc} = \frac{2c}{g} = \frac{2 \cdot 2.99 \times 10^8 \text{m/s}}{9.81 \text{m/s}^2} = 6.10 \times 10^7 \text{ s} \]

The solution is not reasonable since the final velocity of the BMW is \( v_f = g t_{enc} = 2c \), but nothing can go faster than light.
Chapter 2: Kinematics, the math of motion

Let's apply all we learned so far to a real problem: a 25 m ski jump.

View movie at:

www.capca.ucalgary.ca/~rouyed/teaching/Mechanics/phys221.html
Chapter 2: Kinematics, the math of motion

motion diagram → y-position-time graph

Phy221/R. Ouyed
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Translate this into Mathematics:

**Stage 1:** rocky track, let's assume the motion is described by

\[ y(t) = 30 + \frac{5}{4} t^2 (t - 3) \]

for 0 < t < 2, t in seconds, y in meter.
Chapter 2: Kinematics, the math of motion

- How high is he at t=2s?
  Answer: \( y(t=2s) = 25 \text{ m} \)

- What is his velocity at t=2s?
  \[
  v_y(t) = \frac{dy}{dt} = \frac{d}{dt} \left( 30 + \frac{5}{4} t^3 - \frac{15}{4} t^2 \right) = \frac{15}{4} t^2 - \frac{15}{2} t
  \]
  \[\Rightarrow v_y(t=2s) = 0 \text{ m/s}\]

This determines the initial conditions for stage 2.
Stage 2: free fall. Motion is described by
\[ y(t) = y(t_0) + v_y(t_0) \cdot (t - t_0) - \frac{g}{2} \cdot (t - t_0)^2 \]
with \( t_0 = 2 \text{s} \) the start of free fall. We know \( y(t_0) = 25 \text{m} \) and \( v_y(t_0) = 0 \text{ m/s} \)
at \( t=4 \text{s} \) (with \( g \approx 10 \text{ m/s}^2 \)) we find \( y(t=4 \text{s}) \approx 5 \text{ m} \).
Stage 3: inclined plane. Let's assume the snow stopped him so that his initial speed in the plane is 1 m/s. The angle is 45° so that along the direction of the plane

$$s(t) = s(t_0) + v_s(t_0) \cdot (t - t_0) + \frac{g}{2} \sin(45^\circ) \cdot (t - t_0)^2$$

with $t_0 = 4$ s the start of the motion on the plane. The snow stopped him so that $v_s(t_0) = 1$ m/s
Chapter 2: Kinematics, the math of motion

How far has he travelled on the plane after $t=6s$?

$$\Delta s(t) = v_s(t_0) \cdot (t - t_0) + 0.354g \cdot (t - t_0)^2$$

$$\Delta s(6s) = 1\text{ m/s} \cdot 2s + 3.47\text{ m/s}^2 \cdot (2s)^2$$

$$= 15.9\text{ m}$$

How fast is he then?

$$v_s(t) = \frac{ds}{dt}$$

$$= v_s(t_0) + 0.780g \cdot (t - t_0)$$

$$v_s(6s) = 1\frac{\text{ m}}{\text{s}} + 6.95\frac{\text{ m}}{\text{s}^2} \cdot 2s$$

$$= 14.9\frac{\text{ m}}{\text{s}}$$
Chapter 2: Kinematics, the math of motion

Reading Quiz
Chapter 2
Chapter 2: Kinematics, the math of motion

The slope at a point on a position-versus-time graph of an object is

1) the object’s speed at that point.
2) the object’s average velocity at that point.
3) the object’s instantaneous velocity at that point.
4) the object’s acceleration at that point.
5) the distance traveled by the object to that point.
Chapter 2: Kinematics, the math of motion

The area under a velocity-versus-time graph of an object is

1) the object’s speed at that point.
2) the object’s acceleration at that point.
3) the distance traveled by the object.
4) the displacement of the object.
5) This topic was not covered in this chapter.
At the turning point of an object,

1) the instantaneous velocity is zero.
2) the acceleration is zero.
3) Both 1 and 2.
4) Neither 1 nor 2.
5) This topic was not covered in this chapter.
A 1-pound round ball and a 100-pound round ball are placed side-by-side at the top of a frictionless hill. Each is given a very light tap to begin their race to the bottom of the hill. In the absence of air resistance

1) the 1-pound ball wins the race.
2) the 100-pound ball wins the race.
3) the two balls end in a tie.
4) there’s not enough information to determine which ball wins the race.
Chapter 2: Kinematics, the math of motion

Selected Problems
2.2. Model: We will consider Larry to be a particle.

Visualize:

Pictorial representation

Known
- $x_0 = 600$ yds
- $t_0 = 9:05$
- $x_1 = 200$ yds
- $t_1 = 9:07$
- $a = 0$
- $x_2 = 1200$ yds
- $t_2 = 9:10$

Find
- $v_1$
- $v_2$
- $v_{\text{avg}}$
**Chapter 2: Kinematics, the math of motion**

**Solve:** Since Larry’s speed is constant, we can use the following equation to calculate the velocities:

\[ v_s = \frac{s_f - s_i}{t_f - t_i} \]

(a) For the interval from the house to the lamppost:

\[ v_1 = \frac{200 \text{ yd} - 600 \text{ yd}}{9:07 - 9:05} = -200 \text{ yd/min} \]

For the interval from the lamppost to the tree:

\[ v_2 = \frac{1200 \text{ yd} - 200 \text{ yd}}{9:10 - 9:07} = +333 \text{ yd/min} \]

(b) For the average velocity for the entire run:

\[ v_{avg} = \frac{1200 \text{ yd} - 600 \text{ yd}}{9:10 - 9:05} = +120 \text{ yd/min} \]
2.34. **Solve:**  
(a) The velocity is the integral of the acceleration.

\[
v_{1x} = v_{0x} + \int_{t_0}^{t_1} a_x \, dt = 0 \text{ m/s} + \int_{0}^{t_1} (10 - t) \, dt = \left[10t - \frac{1}{2} t^2\right]_0^{t_1} = 10t_1 - \frac{1}{2} t_1^2
\]

The velocity is zero when

\[
v_{1x} = 0 \text{ m/s} = \left(10t_1 - \frac{1}{2} t_1^2\right) = (10 - \frac{1}{2} t_1) \times t_1
\]

\[\Rightarrow t_1 = 0 \text{ s} \quad \text{or} \quad t_1 = 20 \text{ s}
\]

The first solution is the initial condition. Thus the particle’s velocity is again 0 m/s at \( t_1 = 20 \text{ s} \).

(b) Position is the integral of the velocity. At \( t_1 = 20 \text{ s} \), and using \( x_0 = 0 \text{ m} \) at \( t_0 = 0 \text{ s} \), the position is

\[
x_1 = x_0 + \int_{t_0}^{t_1} v_x \, dt = 0 \text{ m} + \int_{0}^{20} \left(10t - \frac{1}{2} t^2\right) \, dt = 5t^2\big|_0^{20} - \frac{1}{6} t^3\big|_0^{20} = 667 \text{ m}
\]
End of Chapter 2

IMPORTANT:

Print a copy of the SUMMARY page (p. 63) and add it here to your lecture notes.

It will save you crucial time when trying to recall: Concepts, Symbols, and Strategies